Linear Programming

1. A linear programming problem deals with the optimization of a/an : (2024)

(A) logarithmic function

(B) linear function

(C) quadratic function

(D) exponential function

Ans. (B) linear function

2. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is : (2024)

(A) 0

(B) 1

(C) 2

(D) 3

Ans. (C) 2

3. Solve the following linear programming problem graphically : (2024)

Maximise $z = 500x + 300y$, subject to constraints

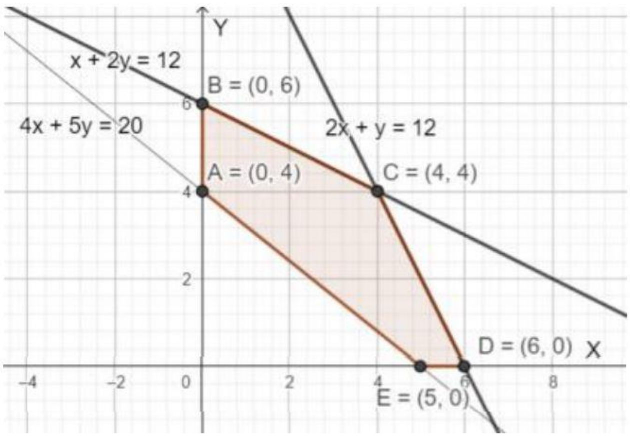
$x + 2y \leq 12$

$2x + y \leq 12$

$4x + 5y \geq 20$

$x \geq 0, y \geq 0$

Ans. Max $z = 500x + 300y$



Corner Point	Z
A (0,4)	1200
B (0,6)	1800
C (4,4)	3200
D (6,0)	3000
E (5,0)	2500

Max $z = 3200$ at $x = 4, y = 4$

Previous Years' CBSE Board Questions

12.2 Linear Programming Problem and its Mathematical Formulation

MCQ

1. Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$?
(a) (–2, 4) (b) (3, 2) (c) (–5, 6) (d) (4, 2)
(2023) U

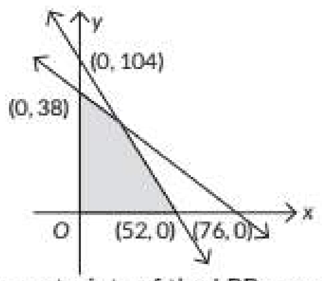
2. The solution set of the inequation $3x + 5y < 7$ is
(a) whole xy-plane except the points lying on the line $3x + 5y = 7$.
(b) whole xy-plane along with the points lying on the line $3x + 5y = 7$.
(c) open half plane containing the origin except the points of line $3x + 5y = 7$.
(d) open half plane not containing the origin.
(2023) U

3. If the corner points of the feasible region of an LPP are (0, 3), (3, 2) and (0, 5), then the minimum value of $Z = 11x + 7y$ is
(a) 21 (b) 33 (c) 14 (d) 35
(Term I, 2021-22) Ev

4. The number of solutions of the system of inequations $x + 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$ is
(a) 0 (b) 2 (c) finite (d) infinite
(Term I, 2021-22) U

5. The maximum value of $Z = 3x + 4y$ subject to the constraints $x \geq 0$, $y \geq 0$ and $x + y \leq 1$ is
(a) 7 (b) 4 (c) 3 (d) 10
(Term I, 2021-22) Ev

6. The feasible region of an LPP is given in the following figure



Then, the constraints of the LPP are $x \geq 0$, $y \geq 0$ and
(a) $2x + y \leq 52$ and $x + 2y \leq 76$
(b) $2x + y \leq 104$ and $x + 2y \leq 76$
(c) $x + 2y \leq 104$ and $2x + y \leq 76$
(d) $x + 2y \leq 104$ and $2x + y \leq 38$
(Term I, 2021-22) Ap

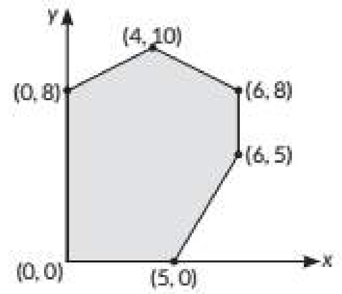
7. If the minimum value of an objective function $Z = ax + by$ occurs at two points (3, 4) and (4, 3) then
(a) $a + b = 0$ (b) $a = b$
(c) $3a = b$ (d) $a = 3b$
(Term I, 2021-22) U

8. For the following LPP, maximise $Z = 3x + 4y$ subject to constraints $x - y \geq -1$, $x \leq 3$, $x \geq 0$, $y \geq 0$ the maximum value is
(a) 0 (b) 4 (c) 25 (d) 30
(Term I, 2021-22) Ap

9. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). If the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both (2, 4) and (4, 0), then
(a) $a = 2b$ (b) $2a = b$
(c) $a = b$ (d) $3a = b$ *(2020)* U

10. In an LPP, if the objective function $z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which z_{max} occurs is
(a) 0 (b) 2 (c) finite (d) infinite
(2020) U

11. The feasible region for an LPP is shown below :
Let $z = 3x - 4y$ be the objective function. Minimum of z occurs at



(a) (0, 0) (b) (0, 8)
(c) (5, 0) (d) (4, 10)
(NCERT Exemplar, 2020) Ap

12. The graph of the inequality $2x + 3y > 6$ is
(a) half plane that contains the origin
(b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.
(c) whole XOY-plane excluding the points on the line $2x + 3y = 6$.
(d) entire XOY-plane. *(2020)* U

13. The objective function of an LPP is
(a) a constant
(b) a linear function to be optimised
(c) an inequality
(d) a quadratic expression *(2020)* R

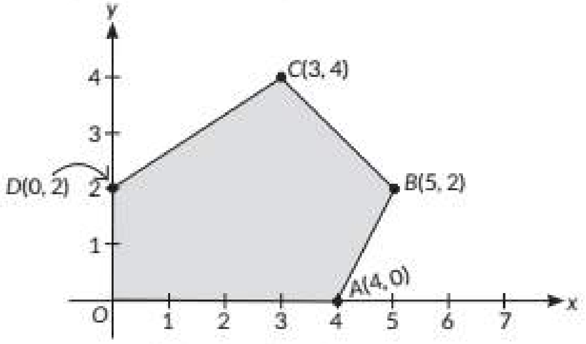
SA II (3 marks)

14. Solve the following linear programming problem graphically :
Maximise $z = -3x - 5y$
Subject to the constraints
$-2x + y \leq 4$
$x + y \geq 3$
$x - 2y \leq 2$,
$x \geq 0, y \geq 0$. *(2023)* EV

LA I (4 marks)

15. Solve the following linear programming problem graphically:
Maximize $z = 3x + 9y$
Subject to constraints
$x + 3y \leq 60$
$x + y \geq 10$
$x \leq y$
$x, y \geq 0$ *(2021)* Ev

16. The corner points of the feasible region determined by the system of linear inequations are as shown below:



Answer each of the following:
(i) Let $z = 13x - 15y$ be the objective function. Find the maximum and minimum values of z and also the corresponding points at which the maximum and minimum values occur.
(ii) Let $z = kx + y$ be the objective function. Find k, if the value of z at A is same as the value of z at B. *(2021)*

17. Solve the following LPP graphically:
Minimize $z = 5x + 7y$
Subject to the constraints
$2x + y \geq 8, x + 2y \geq 10, x, y \geq 0$ *(2020)* Ev

18. Solve the following LPP graphically:
Minimise $Z = 5x + 10y$
Subject to constraints $x + 2y \leq 120, x + y \geq 60,$
$x - 2y \geq 0$ and $x, y \geq 0$
(NCERT Exemplar, Delhi 2017) Ev

19. Maximise $Z = x + 2y$
Subject to the constraints:
$x + 2y \geq 100, 2x - y < 0, 2x + y \leq 200, x, y \geq 0$
Solve the above LPP graphically.
(NCERT, AI 2017) Ev

LA II (5 / 6 marks)

20. Solve the following linear programming problem graphically.
Maximize : $P = 70x + 40y$
Subject to : $3x + 2y \leq 9, 3x + y \leq 9, x \geq 0, y \geq 0.$
(2023) Ev

21. Solve the following linear programming problem graphically.
Minimize : $Z = 60x + 80y$
Subject to constraints:
$3x + 4y \geq 8$
$5x + 2y \geq 11$
$x, y \geq 0$ *(2023)* Cr

22. Find graphically, the maximum value of $z = 2x + 5y$, subject to constraints given below:
$2x + 4y \leq 8, 3x + y \leq 6, x + y \leq 4; x \geq 0, y \geq 0$
(Delhi 2015) Ev

23. Maximise $z = 8x + 9y$ subject to the constraints given below:
$2x + 3y \leq 6, 3x - 2y \leq 6, y \leq 1; x, y \geq 0$
(Foreign 2015) Ev

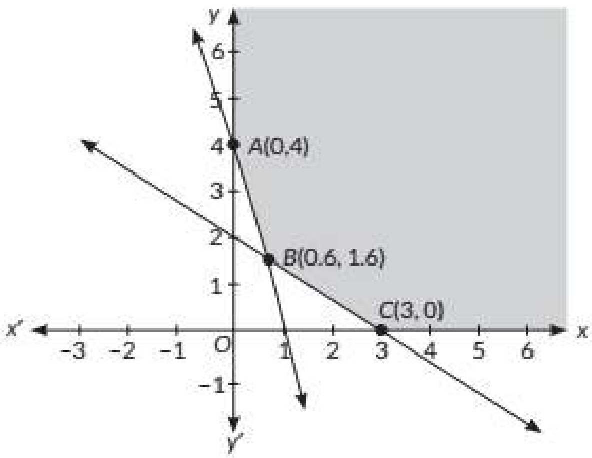
CBSE Sample Questions

12.2 Linear Programming Problem and its Mathematical Formulation

MCQ

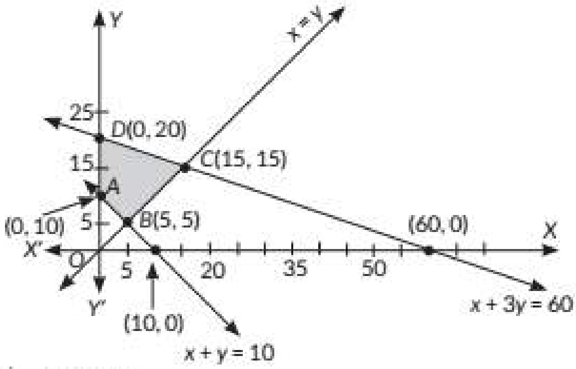
1. The solution set of the inequality $3x + 5y < 4$ is
(a) an open half-plane not containing the origin.
(b) an open half-plane containing the origin.
(c) the whole XY-plane not containing the line $3x + 5y = 4$.
(d) a closed half plane containing the origin.
(2022-23) Ev

2. The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function $Z = 4x + 6y$ occurs at



(a) (0.6, 1.6) only
(b) (3, 0) only
(c) (0.6, 1.6) and (3, 0) only
(d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0) *(2022-23)* Ev

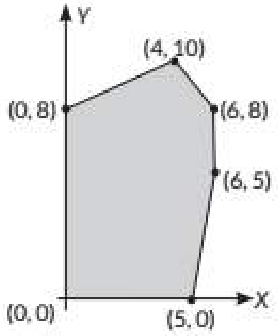
3. Based on the given shaded region as the feasible region in the graph, at which point(s) is the objective function $Z = 3x + 9y$ maximum?



(a) Point B
(b) Point C
(c) Point D
(d) every point on the line segment CD

(Term I, 2021-22) U

4. In the given graph, the feasible region for a LPP is shaded. The objective function $Z = 2x - 3y$, will be minimum at



(a) (4, 10) (b) (6, 8)
(c) (0, 8) (d) (6, 5)

(Term I, 2021-22) Ap

5. A linear programming problem is as follows :
Minimize $Z = 30x + 50y$
Subject to the constraints,
$3x + 5y \geq 15$
$2x + 3y \leq 18$
$x \geq 0, y \geq 0$
In the feasible region, the minimum value of Z occurs at
(a) a unique point
(b) no point
(c) infinitely many points
(d) two points only *(Term I, 2021-22)* U

6. For an objective function $Z = ax + by$, where $a, b > 0$; the corner points of the feasible region determined by a set of constraints (linear inequalities) are (0, 20), (10, 10), (30, 30) and (0, 40). The condition on a and b such that the maximum Z occurs at both the points (30, 30) and (0, 40) is
(a) $b - 3a = 0$ (b) $a = 3b$
(c) $a + 2b = 0$ (d) $2a - b = 0$

(Term I, 2021-22)

7. In a linear programming problem, the constraints on the decision variables x and y are $x - 3y \geq 0$, $y \geq 0$, $0 \leq x \leq 3$. The feasible region
(a) is not in the first quadrant
(b) is bounded in the first quadrant
(c) is unbounded in the first quadrant
(d) does not exist *(Term I, 2021-22)* U

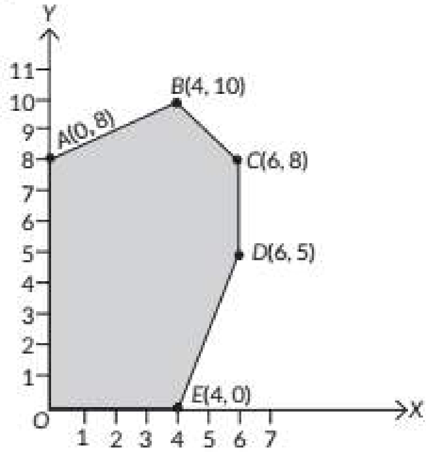
SA II (3 marks)

8. Solve the following Linear Programming Problem graphically:
Maximize $Z = 400x + 300y$ subject to $x + y \leq 200$, $x \leq 40$, $x \geq 20$, $y \geq 0$ *(2022-23)* Ev

LA II (5 / 6 marks)

9. Solve the following linear programming problem (L.P.P) graphically.
Maximize $Z = 3x + y$
Subject to constraints;
$x + 2y \geq 100$
$2x - y \leq 0$
$2x + y \leq 200$
$x, y \geq 0$ *(2020-21)* Ap

10. The corner points of the feasible region determined by the system of linear constraints are as shown below:



Answer each of the following:
(i) Let $Z = 3x - 4y$ be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.
(ii) Let $Z = px + qy$, where $p, q > 0$ be the objective function. Find the condition on p and q so that the maximum value of Z occurs at $B(4,10)$ and $C(6, 8)$. Also mention the number of optimal solutions in this case. *(2020-21)* Ev

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. (d): We have, $2x + y \leq 10$ and $x + 2y \geq 8$

Let us check which of the given points satisfy the given inequation one by one.

(a) $(-2, 4)$

$2 \times (-2) + 4 = -4 + 4 = 0 \leq 10$

and $-2 + 2 \times 4 = -2 + 8 = 6 \not\geq 8$

(b) $(3, 2)$

$2 \times 3 + 2 = 6 + 2 = 8 \leq 10$

$3 + 2 \times 2 = 3 + 4 = 7 \not\geq 8$

(c) $(-5, 6)$

$2 \times (-5) + 6 = -10 + 6 = -4 \leq 10$

$-5 + 2 \times 6 = -5 + 12 = 7 \not\geq 8$

(d) $(4, 2)$

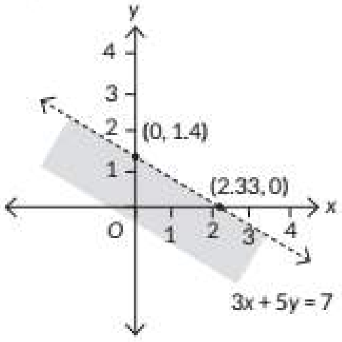
$2 \times 4 + 2 = 10 \leq 10$; $4 + 2 \times 2 = 8 \geq 8$

$\therefore$ (4, 2) satisfy both the inequations.

2. (c): Given inequation is $3x + 5y < 7$

Let us draw the graph of $3x + 5y = 7$

x	0	2.33
y	1.4	0



Substitute, $x = 0$ and $y = 0$ in the inequation, we get

$3(0) + 5(0) < 7$

i.e., $0 < 7$ which is true.

$\therefore$ The solution set of the inequality is an open half plane containing the origin except the points on line $3x + 5y = 7$.

3. (a): Given, $Z = 11x + 7y$

At (0, 3), $Z = 11 \times 0 + 7 \times 3 = 21$

At (3, 2), $Z = 11 \times 3 + 7 \times 2 = 47$

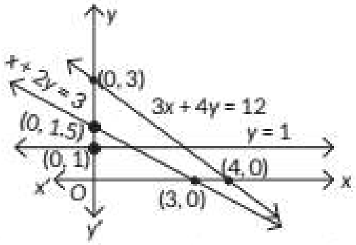
At (0, 5), $Z = 11 \times 0 + 7 \times 5 = 35$

Thus, Z is minimum at (0, 3) and minimum value of Z is 21.

4. (a): Given,

$x + 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$

The graph of given constraints is shown here.



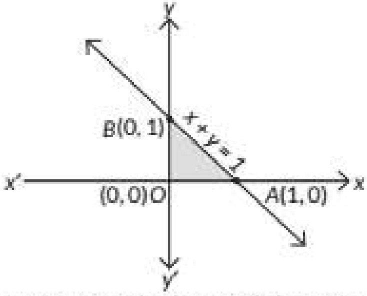
Since, there is no common region, so, no solution exists.

Key Points

- A feasible region is an area defined by a set of coordinates that satisfy a system of inequalities.

5. (b): We have to maximise $Z = 3x + 4y$

Subject to constraints, $x \geq 0$, $y \geq 0$ and $x + y \leq 1$



The shaded portion *OAB* is the feasible region, where *O*(0, 0), *A*(1, 0) and *B*(0, 1) are the corner points.

At *O*(0, 0), $Z = 3 \times 0 + 4 \times 0 = 0$

At *A*(1, 0), $Z = 3 \times 1 + 4 \times 0 = 3$

At *B*(0, 1), $Z = 3 \times 0 + 4 \times 1 = 4$

$\therefore$ Maximum value of Z is 4, which occurs at *B*(0, 1).

Concept Applied

- Any point in the feasible region of a linear programming problem that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

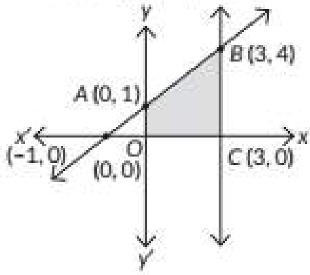
6. (b): Clearly, the pair of points given in graph, and (0, 104); (52, 0) and (0, 38); (76, 0) satisfy the corresponding equations given in option(b) *i.e.*, $2x + y \leq 104$ and $x + 2y \leq 76$.

7. (b): Since, minimum value of $Z = ax + by$ occurs at two points (3, 4) and (4, 3).

$\therefore \quad 3a + 4b = 4a + 3b \Rightarrow a = b$

8. (c): Given, $Z = 3x + 4y$

Subject to constraints, $x - y \geq -1$, $x \leq 3$; $x \geq 0$, $y \geq 0$



The shaded region *OABC* is the feasible region, where corner points are *O*(0, 0), *A*(0, 1), *B*(3, 4) and *C*(3, 0)

At *O* (0, 0), $Z = 3(0) + 4(0) = 0$

At *A* (0, 1), $Z = 3(0) + 4(1) = 4$

At *B* (3, 4), $Z = 3(3) + 4(4) = 25$

At *C* (3, 0), $Z = 3(3) + 4(0) = 9$

$\therefore$ Maximum value of Z is 25, which occurs at *B*(3, 4).

9. (a): Since, maximum value of $z = ax + by$ occurs at both (2, 4) and (4, 0).

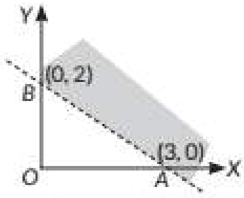
$\therefore \quad 2a + 4b = 4a + 0 \Rightarrow 4b = 2a \Rightarrow 2b = a$

10. (d): In an LPP, if the objective function $z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which z_{max} occurs is infinite.

11. (b) : We know that minimum of objective function occurs at corner points.

Corner points	Value of $z = 3x - 4y$
(0, 0)	0
(5, 0)	15
(6, 5)	−2
(6, 8)	−14
(4, 10)	−28
(0, 8)	−32 ← Minimum

12. (b) : From the graph of inequality $2x + 3y > 6$. It is clear that it does not contain the origin nor the points of the line $2x + 3y = 6$.



13. (b): A linear function to be optimized is called an objective function.

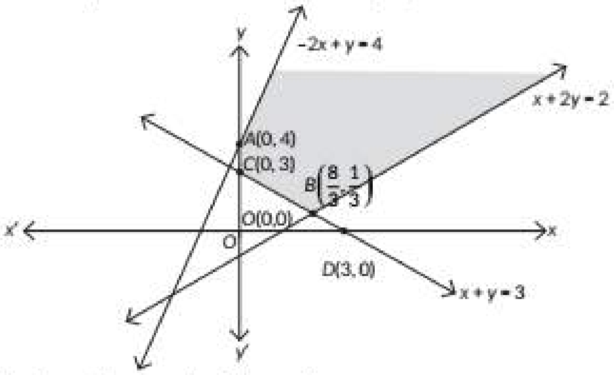
14. We have, maximise $z = -3x - 5y$

Converting the given inequations into equations, we get

$-2x + y = 4$ ...(i)

$x + y = 3$ ...(ii)

$x - 2y = 2$ ...(iii)



We draw the graph of these lines.

As, $x \geq 0, y \geq 0$ so the solution lies in first quadrant.

From graph, corner point of feasible region are $A(0, 4)$, $B(8/3, 1/3)$ and $C(0, 3)$

The value of z at these corner points are shown as :

Corner points	$z = -3x - 5y$	
A(0, 4)	−20	
B(8/3, 1/3)	−29/3	← Maximum
C(0, 3)	−15	

Hence maximum value of $z = \frac{-29}{3}$.

15. We have, maximize $z = 3x + 9y$

Subject to constraints, $x + 3y \leq 60, x + y \geq 10, x \leq y, x, y \geq 0$

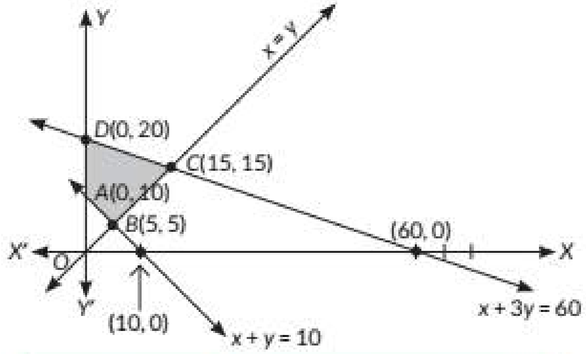
To solve L.P.P. graphically, we convert inequations into equations.

$l_1 : x + 3y = 60, l_2 : x + y = 10, l_3 : x = y, x = 0$ and $y = 0$

l_2 and l_3 intersect at (5, 5). l_1 and l_3 intersect at (15, 15).

The shaded region $ABCD$ is the feasible region and is bounded. The corner points of the feasible region are

$A(0, 10), B(5, 5), C(15, 15)$ and $D(0, 20)$



Corner Points	Value of $z = 3x + 9y$	
A(0, 10)	90	
B(5, 5)	60	
C(15, 15)	180	Maximum (Multiple
D(0, 20)	180	optimal solutions)

The maximum value of Z on the feasible region occurs at the two corner points $C(15, 15)$ and $D(0, 20)$ and it is 180 in each case.

16. (i)

Corner Points	$z = 13x - 15y$
O(0, 0)	0
A(4, 0)	52 (Maximum)
B(5, 2)	35
C(3, 4)	−21
D(0, 2)	−30 (Minimum)

Thus, maximum value of Z is 52 at $A(4, 0)$ and minimum value of Z is −30 at $D(0, 2)$

(ii) Since value of $z = kx + y$ at $A(4, 0)$ is same as the value of Z at $B(5, 2)$.

$\therefore \quad k \cdot 4 + 0 = k \cdot 5 + 2 \Rightarrow 4k = 5k + 2 \Rightarrow k = -2$

17. We have, minimize $z = 5x + 7y$,

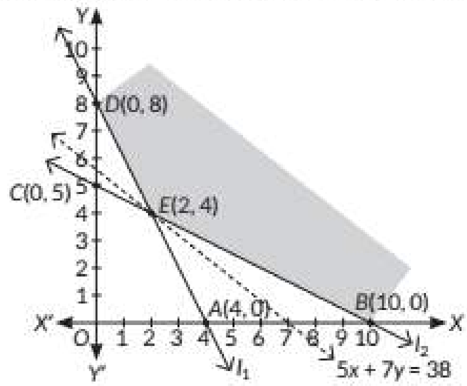
Subject to constraints, $2x + y \geq 8, x + 2y \geq 10, x, y \geq 0$

To solve LPP graphically, we convert inequations into equations.

Now, $l_1 : 2x + y = 8, l_2 : x + 2y = 10$ and $x = 0, y = 0$

l_1 and l_2 intersect at $E(2, 4)$.

Let us draw the graph of these equations as shown below.



The corner points of the feasible region are $D(0, 8)$, $B(10, 0)$ and $E(2, 4)$.

Corner points	Value of $z = 5x + 7y$
$D(0, 8)$	56
$B(10, 0)$	50
$E(2, 4)$	38 (Minimum)

From the table, we find that 38 is the minimum value of z at $E(2, 4)$. Since the region is unbounded, so we draw the graph of inequality $5x + 7y < 38$ to check whether the resulting open half plane has any point common with the feasible region. Since it has no point in common. So, the minimum value of z is obtained at $E(2, 4)$ and the minimum value of $z = 38$.

Answer Tips

- If the region is unbounded, then a maximum or minimum value of the objective function may not exist. If it exists, it must occur at a corner point of region.

18. We have, Minimise $Z = 5x + 10y$,

Subject to constraints :

$x + 2y \le 120$

$x + y \ge 60$

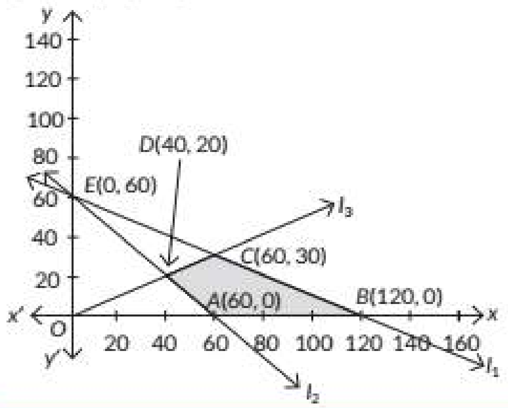
$x - 2y \ge 0$ and $x, y \ge 0$

To solve L.P.P graphically, we convert inequations into equations.

$l_1 : x + 2y = 120$, $l_2 : x + y = 60$, $l_3 : x - 2y = 0$ and $x = 0, y = 0$

l_1 and l_2 intersect at $E(0, 60)$, l_1 and l_3 intersect at $C(60, 30)$, l_2 and l_3 intersect at $D(40, 20)$.

The shaded region $ABCD$ is the feasible region and is bounded. The corner points of the feasible region are $A(60, 0)$, $B(120, 0)$, $C(60, 30)$ and $D(40, 20)$.



Corner points	Value of $Z = 5x + 10y$
$A(60, 0)$	300 ← (Minimum)
$B(120, 0)$	600
$C(60, 30)$	600
$D(40, 20)$	400

Hence, Z is minimum at $A(60, 0)$ *i.e.*, 300.

Commonly Made Mistake

- Remember to convert inequations into equations.

19. Maximise $Z = x + 2y$, Subject to constraints :

$x + 2y \ge 100$, $2x - y < 0$, $2x + y \le 200$ and $x, y \ge 0$.

Converting the inequations into equations, we obtain the lines

$l_1 : x + 2y = 100$ ...(i)

$l_2 : 2x - y = 0$ ...(ii)

$l_3 : 2x + y = 200$ ...(iii)

$l_4 : x = 0$ ...(iv)

and $l_5 : y = 0$ ...(v)

By intercept form, we get

$l_1 : \frac{x}{100} + \frac{y}{50} = 1$

$\Rightarrow$ The line l_1 meets the coordinate axes at (100, 0) and (0, 50).

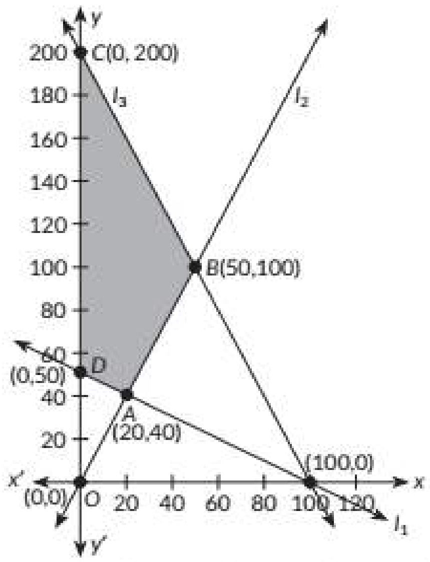
$l_2 : 2x = y$

$\Rightarrow$ The line l_2 passes through origin and cuts l_1 and l_3 at (20, 40) and (50, 100) respectively.

$l_3 : \frac{x}{100} + \frac{y}{200} = 1$

$\Rightarrow$ The line l_3 meets the coordinates axes at (100, 0) and (0, 200).

$l_4 : x = 0$ is the y-axis, $l_5 : y = 0$ is the x-axis



Now, plotting the above points on the graph, we get the feasible region of the LPP as shaded region $ABCD$. The coordinates of the corner points of the feasible region $ABCD$ are $A(20, 40)$, $B(50, 100)$, $C(0, 200)$, $D(0, 50)$.

Now, $Z_A = 20 + 2 \times 40 = 100$

$Z_B = 50 + 2 \times 100 = 250$, $Z_C = 0 + 2 \times 200 = 400$

$Z_D = 0 + 2 \times 50 = 100$

$\therefore$ Z is maximum at $C(0, 200)$ and having value 400.

20. We have, maximize $P = 70x + 40y$

Subject to : $3x + 2y \le 9$

$3x + y \le 9$

$x \ge 0, y \ge 0$

Convert all inequations into equation, we get

$3x + 2y = 9$ ... (i)

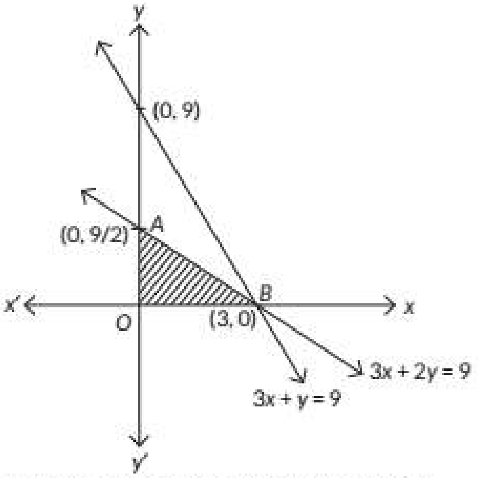
$3x + y = 9$ ... (ii)

$x = 0$ and $y = 0$

Solving (i) and (ii), we get

$x = 3, y = 0$

So, point of intersection of equation (i) and (ii) are (3, 0).

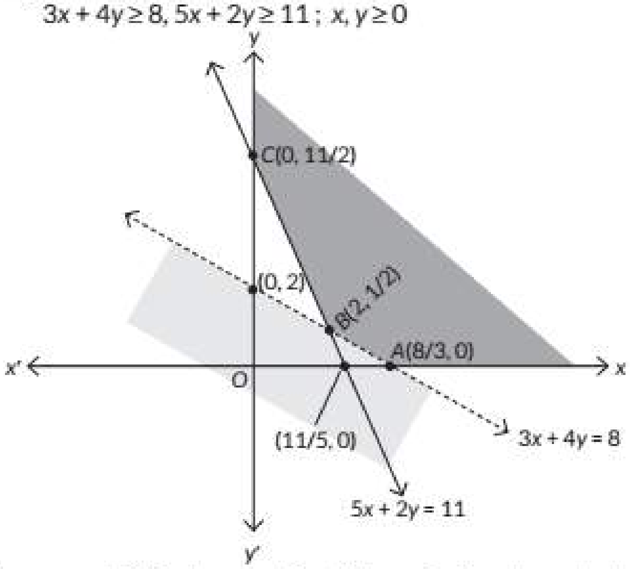


The given shaded region is the feasible region.
The corner points of the feasible region are $O(0, 0)$, $A(0, 9/2)$ and $B(3, 0)$.

Corner points	Value of $p = 70x + 40y$
$O(0, 0)$	$70 \times 0 + 40 \times 0 = 0$
$A(0, 9/2)$	$70 \times 0 + 40 \times \frac{9}{2} = 180$
$B(3, 0)$	$70 \times 3 + 40 \times 0 = 210$ (maximum)

So, P is maximum at point $B(3, 0)$.

21. We have, $\min z = 60x + 80y$;
Subject to constraints;
$3x + 4y \geq 8,\ 5x + 2y \geq 11;\ x, y \geq 0$



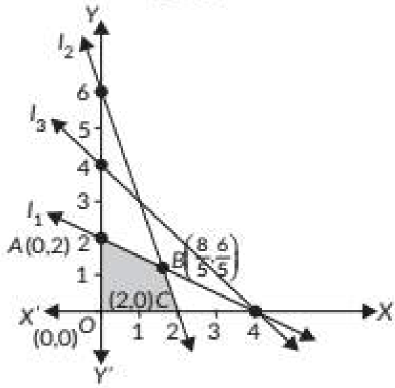
From graph, it is clear that feasible region is unbounded.
The corner point of the feasible region are $A(8/3, 0)$, $B(2, 1/2)$ and $C(0, 11/2)$.
The value of Z at these corner points are as follows :

Corner Points	$Z = 60x + 80y$	
$A(8/3, 0)$	160	(Minimum)
$B(2, 1/2)$	160	
$C(0, 11/2)$	440	

As the feasible region is unbounded,
$\therefore$ 160 may or may not be the minimum value of Z.
So, we graph the inequality $60x + 80y < 160$ *i.e.*, $3x + 4y < 8$ and check whether the resulting half plane has points in common with the feasible region or not.
From graph, it can be seen that feasible region has no common point with $3x + 4y < 8$
$\therefore$ Minimum value of Z is 160 at the line joining the points $(8/3, 0)$ and $(2, 1/2)$.

22. Let $l_1 : 2x + 4y = 8, l_2 : 3x + y = 6, l_3 : x + y = 4; x = 0, y = 0$

Solving l_1 and l_2 we get $B\left(\frac{8}{5}, \frac{6}{5}\right)$



Shaded portion $OABC$ is the feasible region, where coordinates of the corner points are $O(0, 0)$, $A(0, 2)$, $B\left(\frac{8}{5}, \frac{6}{5}\right)$, $C(2, 0)$

The value of objective function at these points are :

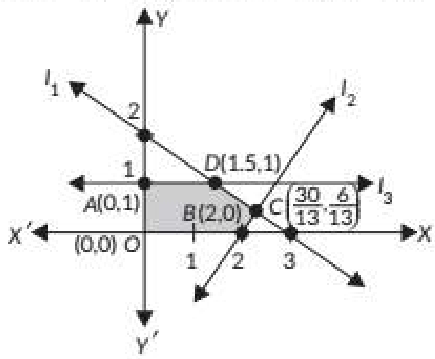
Corner points	Value of the objective function $z = 2x + 5y$
$O(0, 0)$	$2 \times 0 + 5 \times 0 = 0$
$A(0, 2)$	$2 \times 0 + 5 \times 2 = 10$ (Maximum)
$B\left(\frac{8}{5}, \frac{6}{5}\right)$	$2\times\frac{8}{5}+5\times\frac{6}{5}=9.2$
$C(2, 0)$	$2 \times 2 + 5 \times 0 = 4$

$\therefore$ The maximum value of z is 10, which is at $A(0, 2)$.

Concept Applied

- If the region is bounded then the objective function Z has both maximum and minimum value of region.

23. Let $l_1 : 2x + 3y = 6, l_2 : 3x - 2y = 6, l_3 : y = 1; x = 0, y = 0$



Solving l_1 and l_3, we get D (1.5, 1)

Solving l_1 and l_2, we get $C\left(\frac{30}{13}, \frac{6}{13}\right)$

Shaded portion *OADCB* is the feasible region, where coordinates of the corner points are $O(0, 0)$, $A(0, 1)$, $D(1.5, 1)$, $C\left(\frac{30}{13}, \frac{6}{13}\right)$, $B(2, 0)$.

The value of the objective function at these points are :

Corner points	Value of the objective function $z = 8x + 9y$
$O(0, 0)$	$8 \times 0 + 9 \times 0 = 0$
$A(0, 1)$	$8 \times 0 + 9 \times 1 = 9$
$D(1.5, 1)$	$8 \times 1.5 + 9 \times 1 = 21$
$C\left(\frac{30}{13}, \frac{6}{13}\right)$	$8\times\frac{30}{13}+9\times\frac{6}{13}=22.6$ (Maximum)
$B(2, 0)$	$8 \times 2 + 9 \times 0 = 16$

The maximum value of z is 22.6, which is at $C\left(\frac{30}{13}, \frac{6}{13}\right)$.

Commonly Made Mistake

- Remember the difference between feasible solutions and infeasible solutions.

CBSE Sample Questions

1. **(b):** The strict inequality represents an open half plane and it contains the origin, as (0, 0) satisfies it. **(1)**

2. **(d):** The minimum value of the objective function occurs at two adjacent corner points (0.6, 1.6) and (3, 0) and there is no point in the half plane $4x + 6y < 12$ in common with the feasible region.
So, the minimum value occurs at every point of the line-segment joining the two points. **(1)**

3. **(d):** We have,

Corner points	Value of $Z = 3x + 9y$
$A(0, 10)$	$3 \times 0 + 9 \times 10 = 90$
$B(5, 5)$	$3 \times 5 + 9 \times 5 = 60$
$C(15, 15)$	$3 \times 15 + 9 \times 15 = 180$ (Maximum)
$D(0, 20)$	$3 \times 0 + 9 \times 20 = 180$ (Maximum)

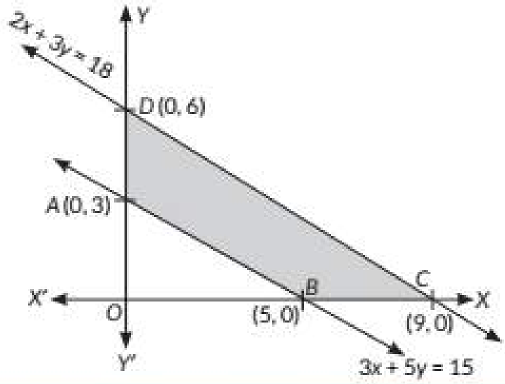
$\because$ Z is maximum at $C(15, 15)$ and $D(0, 20)$.
$\therefore$ Z is maximum at every point on the line joining *CD*. **(1)**

4. **(c):** We have,

Corner points	Value of $Z = 2x - 3y$
(0, 0)	$2 \times 0 - 3 \times 0 = 0$
(0, 8)	$2 \times 0 - 3 \times 8 = -24$ (Minimum)
(4, 10)	$2 \times 4 - 3 \times 10 = -22$
(6, 8)	$2 \times 6 - 3 \times 8 = -12$
(6, 5)	$2 \times 6 - 3 \times 5 = -3$
(5, 0)	$2 \times 5 - 3 \times 0 = 10$

$\therefore$ Value of Z is minimum at (0, 8). **(1)**

5. **(c):** Here, the feasible region is shaded.

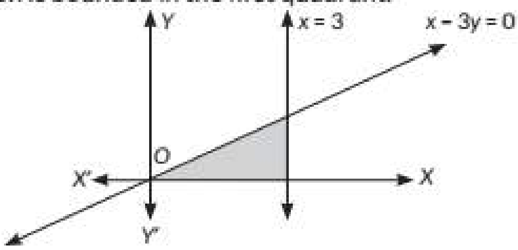


Corner points	Value of $Z = 30x + 50y$
$A(0, 3)$	$30 \times 0 + 50 \times 3 = 150$ (Minimum)
$B(5, 0)$	$30 \times 5 + 50 \times 0 = 150$ (Minimum)
$C(9, 0)$	$30 \times 9 + 50 \times 0 = 270$
$D(0, 6)$	$30 \times 0 + 50 \times 6 = 300$

Since, minimum value of Z occurs at both A and B. So, Z is minimum at every point on the line joining *AB*. So, minimum value of Z occurs at infinitely many points. **(1)**

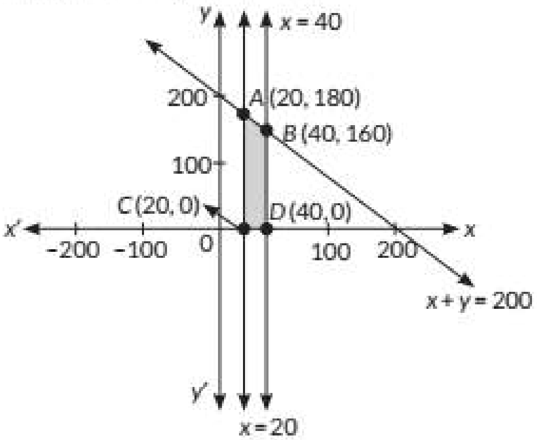
6. **(a):** As, Z is maximum at (30, 30) and (0, 40).
$\Rightarrow$ $30a + 30b = 40b \Rightarrow b - 3a = 0$ **(1)**

7. **(b):** From the graph, we can say that the feasible region is bounded in the first quadrant.



(1)

8. We have $Z = 400x + 300y$ subject to
$x + y \leq 200$, $x \leq 40$, $x \geq 20$, $y \geq 0$
The corner points of the feasible region are $C(20, 0)$, $D(40, 0)$, $B(40, 1600)$, $A(20, 180)$



(1)

Corner points	$Z = 400x + 300y$
$C(20, 0)$	8000
$D(40, 0)$	16000
$B(40, 160)$	64000
$A(20, 180)$	62000

(1)

Maximum profit occurs at $x = 40, y = 160$

and the maximum profit = ₹ 64,000 **(1)**

9. Maximize $Z = 3x + y$

Subject to constraints

$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

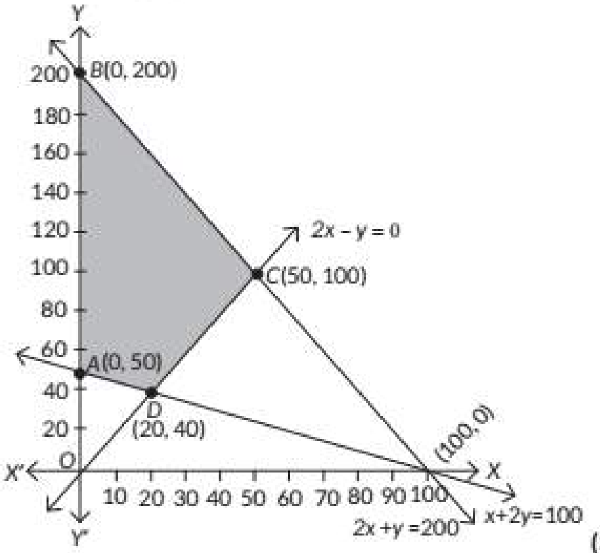
$x \geq 0, y \geq 0$

Converting the given inequations into equations, we get

$x + 2y = 100$ ... (i) $2x - y = 0$ ... (ii)

$2x + y = 200$ ... (iii)

Now, draw the graphs of (i), (ii) and (iii).



(3)

The feasible region is shaded region and corner points are $A(0, 50)$, $B(0, 200)$, $C(50, 100)$ and $D(20, 40)$. **(1)**

The values of Z at corner points are shown in the following table:

Corner points	$Z = 3x + y$
A (0, 50)	50
B (0, 200)	200
C (50, 100)	250 (Maximum)
D (20, 40)	100

Thus, maximum value of Z is 250 at $x = 50, y = 100$. **(1)**

10. (i)

Corner points	$Z = 3x - 4y$
O (0, 0)	0
A (0, 8)	–32 (Minimum)
B (4, 10)	–28
C (6, 8)	–14
D (6, 5)	–2
E (4, 0)	12 (Maximum)

(1½)

Thus, maximum value of Z is 12 at $E(4, 0)$.

and minimum value of Z is –32 at $A(0, 8)$. **(1)**

(ii) Since maximum value of Z occurs at $B(4, 10)$ and $C(6, 8)$.

$\therefore \quad 4p + 10q = 6p + 8q \Rightarrow 2q = 2p \Rightarrow p = q$ **(2)**

Number of optional solutions are infinite.

[$\because$ Every point on the line segment BC joining the two corner points B and C also give the same maximum value] **(1/2)**